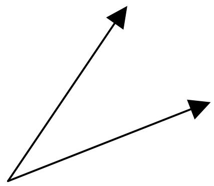
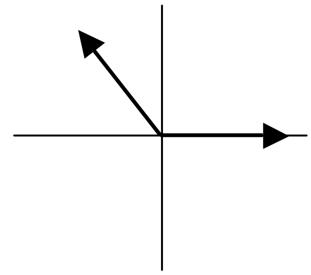
HON PRECALCULUS Name:________________________________

Introduction to Trigonometry Notes

I. **Angle**



In a coordinate system:

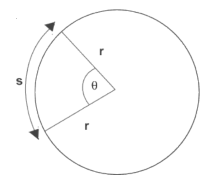


II. **Angle measurement** in a coordinate system (**standard position**)

A. Always measured from **initial ray** to **terminal ray**- initial ray is on positive x-axis; angles with the same initial and terminal ray are called **coterminal**

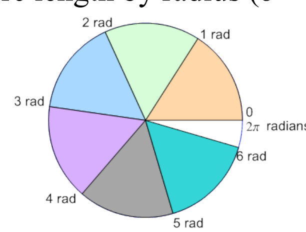
B. Clockwise = **negative** angle; counter-clockwise = **positive** angle (right-hand rule)

C. Two basic units of measurement:



1. **Degrees** – one complete revolution = 360°

2. **Radians** – one complete revolution = 2π radians
   the radian measure of an angle is calculated by dividing the arc length by radius ($\theta = s/r$)



D. **Conversion**

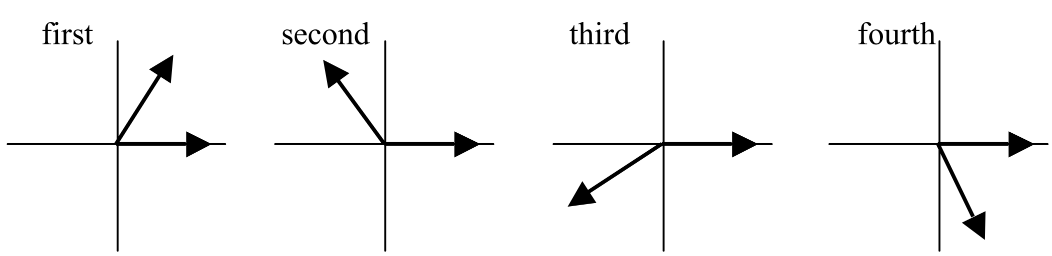
The above facts (1 & 2) set up the relationship that there are π radians in 180° (NOT $\pi = 180$) which can be turned into two useful ratios:

to convert an angle that is in degrees to one in radians, multiply degrees by $\dfrac{\pi}{180}$

to convert an angle that is in radians to one in degrees, multiply radians by $\dfrac{180}{\pi}$

Alternatively, make a proportion and solve.

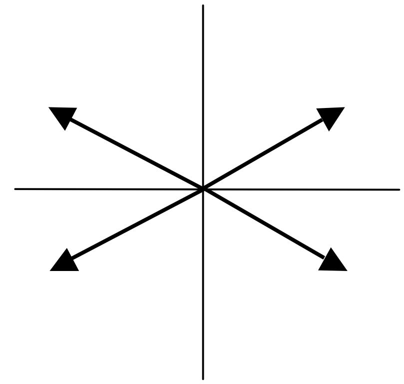
E. **Quadrant angles** – defined by the terminal ray



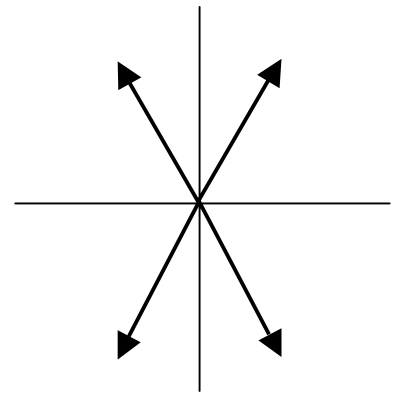
III. **Special Angle Sets**

A. 30˚-type angles: 30°, 150°, 210°, 330° $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

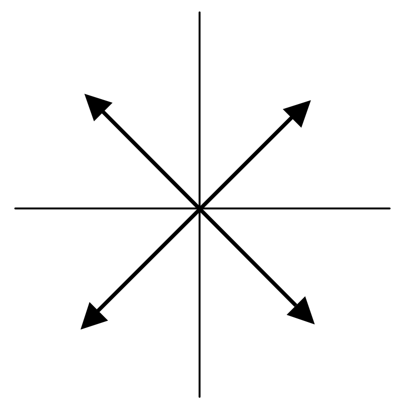
(**reference angle**)



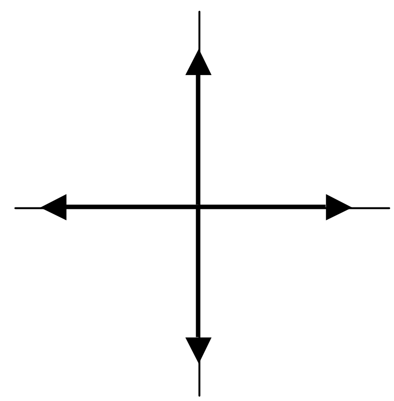
B. 60˚-type angles: 60°, 120°, 240°, 300° $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$



C. 45˚-type angles: 45°, 135°, 225°, 315° $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



D. Axis angles: 0°, 90°, 180°, 270°, 360° $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

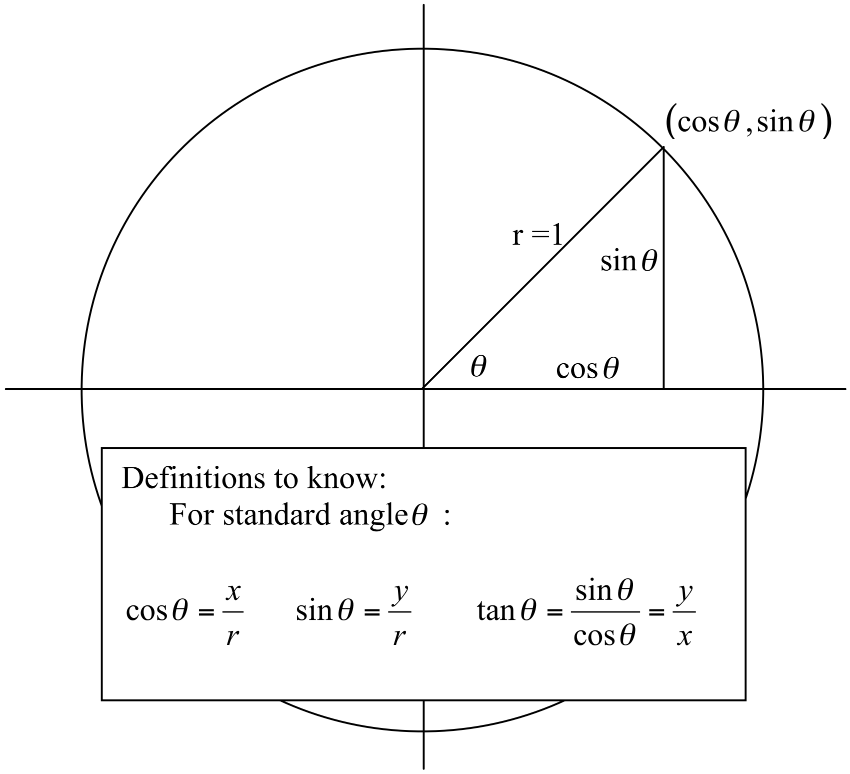


IV. **Unit Circle** – the unit circle is just an aid to help you learn about the relationships that the functions sine and cosine have to all angles (and we will use this for tangent also).

The definitions of the 3 basic trig functions are given in the box below. For a circle of radius one, a unit circle, the "*r*" in the definitions disappears (because it has a value of 1)

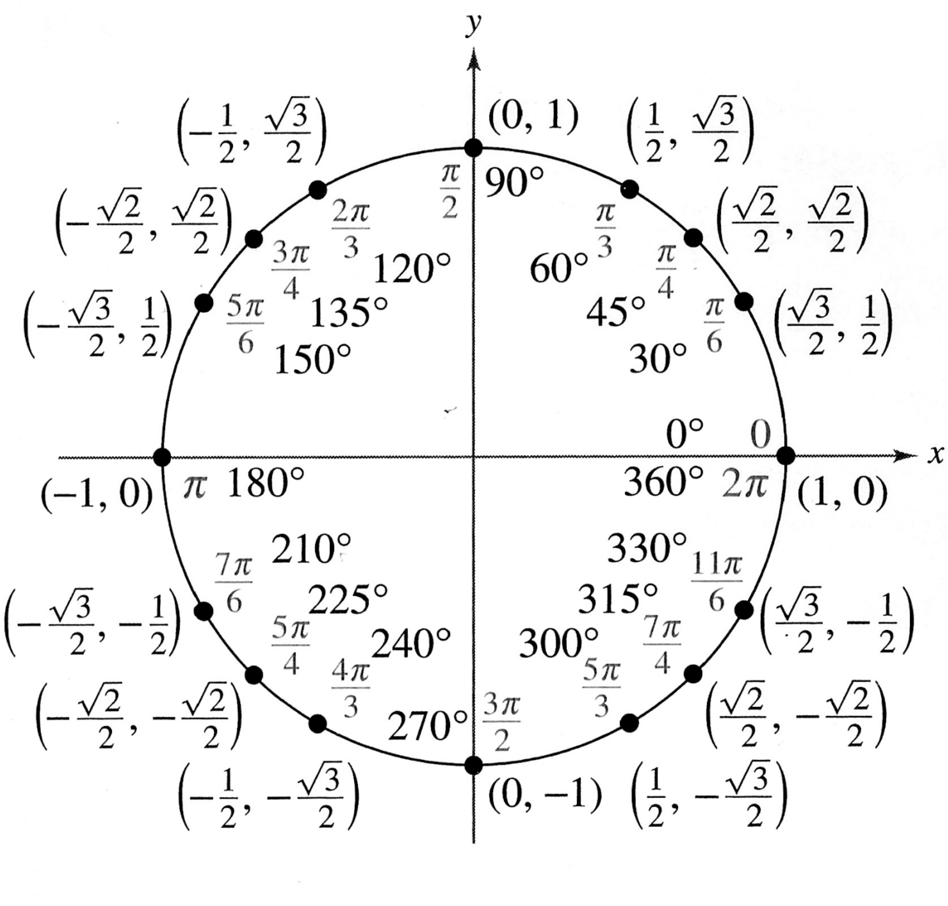
Any point on the circle, which would have the coordinates (x, y) now can be transformed into trigonometric coordinates $(\cos\theta, \sin\theta)$.

These coordinates still represent the same point, one measure is *x* and *y* distance, the other measure is rotating a ray through an angle (θ) and then going out a distance on that ray (r).



Definitions to know:
For standard angle θ :

$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$



V. Trigonometric Values For The Special Angle Set (this is what the unit circle helps you learn)

θ (deg)	θ (rad)	$\cos\theta$	$\sin\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
0°	0	1	0	0	1	undef	undef
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$	2	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	2	$\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
90°	$\frac{\pi}{2}$	0	1	undef	undef	1	0
120°	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	-2	$\frac{2}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$
135°	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
150°	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{1}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$	2	$-\sqrt{3}$
180°	π	-1	0	0	-1	undef	undef
210°	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{3}$
225°	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240°	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\sqrt{3}$	-2	$-\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
270°	$\frac{3\pi}{2}$	0	-1	undef	undef	-1	0
300°	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	2	$-\frac{2}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$
315°	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
330°	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$	-2	$-\sqrt{3}$
360°	2π	1	0	0	1	undef	undef

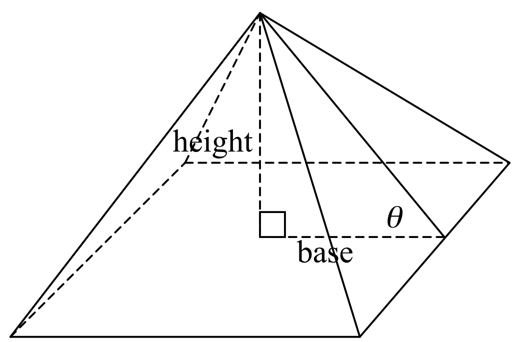
A Brief Story of Trigonometry

Trigonometry, the mathematics that is the relation between linear measurements and angular measurements, has been around for quite a while.

In 1858, Scottish lawyer and antiquarian, Henry Rhind, traveled to Egypt and purchased an ancient papyrus scroll. This scroll is known now as the Rhind Papyrus.

The Rhind Papyrus is over 3800 years old (Rhind did not know this when he purchased it, and, I would speculate that the seller didn't either) and it dates back to around 1800 BC. It is a document of 84 mathematics problems. These problems were used to educate individuals in the important mathematics of the time – so, in essence, the Rhind Papyrus was an early textbook.

Of particular interest is a set of problems that illustrate the mathematics of the construction of pyramids. In this problem set, there is a measurement referenced called the "seked" which is the ratio of two linear measurements. As it turns out, when analyzed in modern mathematical language, the seked is based on the ratio of the base to the height of a right triangle. In modern terms, we refer to this as the cotangent of the angle that is formed in the right triangle shown in the diagram below.



$$\text{seked} = \cot\theta = \frac{\text{base}}{\text{height}}$$

The clear conclusion from this is that the early Egyptian engineers used a crude, but effective, form of trigonometry.

The ancient Babylonians devised a counting system based on 60 (called the sexagesmal number system – later, Arabic scholars would change this to base-10, the number system we use today). It is thought that the number 60 was used because of its relation to 360 which is the approximate number of days in the year (remember, the Gregorian Calendar was centuries away) and the ultimate relation that the year had to agricultural cycles. The division of a year into 360 parts also became the basis for dividing a circle into 360 degrees – you can see the connection between a year and a circle.

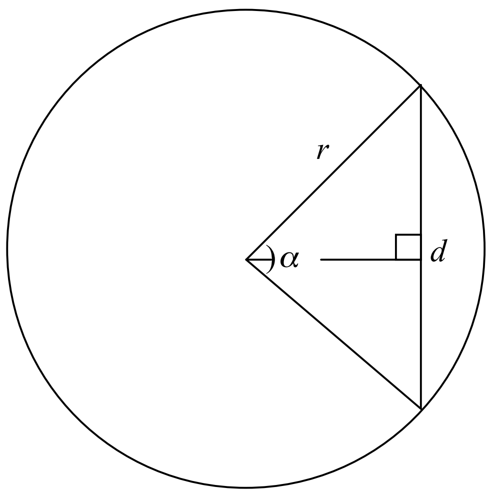
The origin of the word "degree" came later from the Greeks who used the word "moira" to denote the 360th part of a circle. Later, the Arabic translation of this word was "daraja". In Latin, daraja became "de graus" which eventually became anglicized as the word degree.

Greek mathematicians called one sixtieth $\left(\frac{1}{60}\right)$ of a degree "the first part", which in Latin is "pars minuta prima" – shortened to "minute" – and one sixtieth of that ($\frac{1}{3600}$ of a degree) "the second part", which in Latin is "pars minuta secunda" – shortened to "second".

In 1871 (AD), James Thompson, brother of Lord Kelvin, first used the concept of a "radian" to describe an angle measure as an alternative to the use of degrees. His method was quickly adopted by mathematicians of the day and has prevailed ever since.

Trigonometry, in the modern sense, began with Hipparchus (190-120 BC), who is said to be the greatest astronomer of antiquity. He identified 100 stars and cataloged their celestial latitude and longitude – a feat that would require an extensive understanding of the branch of mathematics that we call trigonometry, not to mention many, many sleepless nights! He devised a set of tables to help him accomplish this task – tables that are really an ancient form of trig tables.

Later, Ptolemy (85-165 AD), one of the greatest teachers of the ancient world, wrote a book called The Almagest that was a summary of the mathematics of astronomy. In it exists a *Table of Chords*. The length of a chord of a circle of known radius is related to that radius by the size of the radius and the measure of the central angle using the "sine" function:



$$d = 2r\sin\frac{\alpha}{2}$$

The calculations he made were extraordinarily accurate. For example, a central angle of 7 degrees corresponds to a chord length of 7.32583 in Ptolemy's table. Using modern mathematics and a calculator the actual chord length would measure 7.32582. This is an impressive level of accuracy for a document that was written just under 2000 years ago (no calculators or computers then…).

Hindu mathematics played a role as well. An ancient Hindu text called Aryabhatiya, written by Aryabhata, is the earliest known Hindu text on pure mathematics (c. 510 AD). In it, the word "ardha-jya" is used, which means "half chord". This was eventually turned around to "jya-ardha" (chord half). It was later shortened to "jya" or "jiva". When Arab scholars translated this work into Arabic, the word "jiva" was retained but it was pronounced "jiba" or "jaib", the second form of which finally stuck. Jaib, in Arabic, means bosom, fold or bay. Later, when Latin scholars translated Arabic works then used the meaning of the word for their translation and made "jaib" the word "sinus", which also means bosom, bay or curve.

The word "sinus" was eventually shortened to "sine" and saw its first use in publication by an Englishman, Edmund Gunter, around 1600.

Credit: Trigonometric Delights, Eli Maor, Princeton University Press, 1998